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Computational aspects of prospect theory with asset pricing applications

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Abstract We develop an algorithm to compute asset allocations for Kahneman and Tversky's (*Econometrica, 47*(2), 263–291, 1979) prospect theory. An application to benchmark data as in Fama and French (*Journal of Financial Economics, 47*(2),427–465, 1992) shows that the equity premium puzzle is resolved for parameter values similar to those found in the laboratory experiments of Kahneman and Tversky (*Econometrica, 47*(2), 263–291, 1979). While previous studies like Benartzi and Thaler (*The Quarterly Journal of Economics, 110*(1), 73–92, 1995), Barberis, Huang and Santos (*The Quarterly Journal of Economics, 116*(1), 1–53, 2001), and Grüne and Semmler (*Asset prices and loss aversion*, Germany, Mimeo Bielefeld University, 2005) focussed on dynamic aspects of asset pricing but only used loss aversion to explain the equity premium puzzle our paper explains the unconditional moments of asset pricing by a static two-period optimization problem. However, we incorporate asymmetric risk aversion. Our approach allows reducing the degree of loss aversion from 2.353

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to 2.25, which is the value found by Tversky and Kahneman (*Journal of Risk and Uncertainty, 5*, 297–323, 1992) while increasing the risk aversion from 1 to 0.894, which is a slightly higher value than the 0.88 found by Tversky and Kahneman (*Journal of Risk and Uncertainty, 5*, 297–323, 1992). The equivalence of these parameter settings is robust to incorporating the size and the value portfolios of Fama and French (*Journal of Finance, 47*(2), 427–465, 1992). However, the optimal prospect theory portfolios found on this larger set of assets differ drastically from the optimal mean-variance portfolio.

Keywords Prospect theory · Asset pricing · Equity premium puzzle · Global optimization · Non–smooth problems · Numerical algorithms

1 Introduction

Kahneman and Tversky's (1979) prospect theory has been suggested to explain decisions under risk observed in laboratory experiments. This path breaking observation has led researchers to rethink many areas in economics and in particular in finance. However, applying prospect theory to finance is quite a challenge at least numerically since the value function suggested by Tversky and Kahneman (1992) is non-differentiable and non-concave. In this paper we develop an algorithm to overcome these difficulties so that we can compute prospect theory asset allocations. As a first application we reconsider the equity premium puzzle from a prospect theory perspective.

A large equity premium is one of the more robust findings in financial economics (Mehra & Prescott, 1985). On US-data, for example, over the period of 1802 to 1998, Siegel (1998) reports an excess return of US Stocks over US Bonds of about 7–8% p.a.. Similar results have been found for other periods and across other countries. This empirical finding is puzzling because it is hard to reconcile with plausible parameter values for agents' risk aversion in the standard consumption based asset pricing model originating from Lucas (1978). In the Lucas model asset prices are explained by the intertemporal optimization of a representative agent. The equity premium puzzle is the observation that the unconditional moments of asset returns cannot be explained by a low coefficient of risk aversion. In this model the volatility of consumption, which, as Hansen and Jagannathan (1991) have shown, is an upper bound for the equity premium, is not found to be sufficiently high to allow for the large equity premium.

The huge number of solutions that have been suggested to resolve the equity premium puzzle is another puzzling aspect of the equity premium. It is impossible to review this literature in a few words without being accused for serious omissions. Therefore, we only highlight a few suggested solutions that relate most closely to our paper. For a recent comprehensive treatment of all suggested solutions see the recent Handbook of Finance on this topic edited by Mehra (2006). One line of research points out that the Hansen and Jagannathan bound is increased by choosing more appropriate proxies for consumption that include fluctuations in financial wealth (Lettau & Ludvigson, 2001), fluctuations in housing wealth (Piazzesi, Schneider, & Tuzel, 2007) or other such proxies. Another line of research points out that introducing more links in the utility function between consumption of different periods can also resolve the equity premium puzzle (Constantinides, 1990; Constantinides, Donaldson, & Mehra (2005)). Yet a different strategy is to generate extra fluctuations by including behavioral aspects like myopic loss aversion (Barberis, Huang, & Santos, 2001; Benartzi & Thaler, 1995; Grüne & Semmler, 2005).

Our paper follows the behavioral approach to the equity premium puzzle. This explanation of the equity premium puzzle is based on two main ideas, myopia and loss aversion. First, investors evaluate risky assets by the gains and losses on a short horizon but not by the final wealth the investor achieves. For this reason instead of applying a utility function to consumption data Benartzi and Thaler (1995) apply a value function to asset return data describing the gains and losses. Second, these papers use the fact that losses loom more than gains, i.e., that the value function is kinked at the reference point which divides returns into gains and losses. To this explanation we add a third aspect, asymmetric risk aversion. Kahneman and Tversky (1979) found that agents are risk averse in the gain region but risk seeking in the loss region.

The value function suggested by Tversky and Kahneman (1992) is the following piecewise power function:

$$v(x) = \begin{cases} x^{\alpha} & \text{for } x \geq 0 \\ -\beta(-x)^{\alpha} & \text{for } x < 0, \end{cases} \quad \text{where } \alpha \approx 0.88 \quad \text{and} \quad \beta \approx 2.25, \tag{1}$$

which is concave for gains ($x > 0$) and convex for losses ($x < 0$). Moreover, note that the value function is kinked at $x = 0$.

While the studies of Benartzi and Thaler (1995), Barberis et al. (2001), and Grüne and Semmler (2005) only used myopic loss aversion to explain the equity premium puzzle, i.e., they define the value function on changes in wealth and they set $\alpha = 1$, our paper incorporates the other important aspect of prospect theory: asymmetric risk aversion, i.e., we allow for $\alpha < 1$. A priori it is unclear in which direction the inclusion of asymmetric risk aversion will drive the result because the value function becomes more concave in some region and more convex in the other region. Therefore, we analyze standard annual US asset market data from 1927 to 2002 that has been grouped into benchmark portfolios according to the methodology of Fama and French (1992, 1993).[1] We set the reference point for the net-return data of Fama and French (1993) equal to 0, assuming that obtaining the risk free rate is already seen as a success of the investment. Our extension to asymmetric risk aversion allows reducing the degree of loss aversion from 2.353 to 2.25 while increasing the risk aversion from 1 to 0.894. Hence, the parameter values we find are more similar to those found by Tversky and Kahneman (1992), which were 2.25 for the loss aversion and 0.88 for the risk aversion. The degree to which these parameter values

[1] We are most grateful to Thierry Post for having made available this excellent data set.

coincide with those found in the laboratory is amazing because the former are determined on aggregate financial data while the latter have been determined by investigating individual decision problems. However, as we explain below, due to a robustness problem when maximizing prospect theory value functions these specific values should also not be overemphasized.

Note however that in contrast to Barberis et al., (2001) we do only study a static two-period problem. That is to say, we do not deal with changes of the reference point or changes of risk aversion due to gains and losses. Neither we do investigate whether successive gains and losses are integrated or separated.

One reason for the omission of asymmetric risk aversion in the previous studies may be that prospect theory including this feature becomes much more difficult to apply. Since the value function is then convex for losses the objective function is no longer quasi–concave and the first order condition may only describe local optima. Indeed on our data we found local optima for prospect theory with asymmetric risk aversion. The computational part of this paper describes a fast, efficient and robust method to nevertheless compute optimal prospect theory portfolios. Finally, we analyze the equivalence of the parameter settings mentioned above on the complete data set now including the Fama and French (1992) size and value sorted portfolios. It is found that the optimal prospect theory portfolios are still quite similar and that they do differ drastically from the optimal mean-variance portfolio.

The rest of the paper is organized as follows. Section 2 introduces the model setup and the algorithm for determining prospect theory optimal allocations. Section 3 presents an asset pricing application dealing with the equity premium puzzle. Section 4 concludes.

2 Computational aspects of prospect theory

In this section we analyze the prospect theory of Kahneman and Tversky (1979) from a computational point of view. One computational difficulty is loss aversion which leads to a non-differentiability at the reference point. In the case of a piecewise linear value function the differentiability problem can however easily be dealt with because for $\alpha = 1$ maximization of the prospect utility amounts to solving a linear program. For the more general case we will evoke some smoothing techniques to get around the non–differentiability. The next problem that arises for $\alpha < 1$ is that the objective function in not quasi–concave since the value function is convex for losses and concave for gains. As an effect local optima can arise. We solve this problem by choosing randomly selected starting points for our algorithm.

2.1 The general computational problem

The Kahneman–Tversky value function (1) can also be formulated in the following compact form

$$v(x) = (x^{+})^{\alpha} - \beta\,(x^{-})^{\alpha} \tag{2}$$

with $x^+ = \max\{0, x\}$ and $x^- = \max\{0, -x\}$ denoting the positive and negative parts of the real number x, respectively. Figure 1 displays $v(x)$ with the parameter selection $\alpha = 0.88$ and $\beta = 2.25$. For formulating the asset allocation problem we consider n assets with asset weights λ_j, $j = 1, \ldots, n$, for which we have the constraint $\sum_{j=1}^{n} \lambda_j = 1$ and will focus on the case $\lambda_j \geq 0 \ \forall j$, that means, we assume that short sales are not allowed. Regarding data for the asset returns, we presuppose that scenarios r_j^s are given for the net return of asset j in scenario s, $j = 1, \ldots, n$, $s = 1, \ldots, S$. The portfolio return in scenario s will be $x^s := \sum_{j=1}^{n} r_j^s \lambda_j = (r^s)^{\mathrm{T}} \lambda$ with r^s standing for the vector of portfolio returns in scenario s and λ denoting the vector of the asset weights. For the sake of simplicity of presentation, we assume also that the return scenarios r^s are equally probable.[2] Then, the prospect theory asset allocation problem consists of maximizing the objective function[3]

$$V(x^1, \ldots, x^S) := \frac{1}{S} \sum_{s=1}^{S} v(x^s) = \frac{1}{S} \sum_{s=1}^{S} \left[\left((x^s)^+\right)^\alpha - \beta \left((x^s)^-\right)^\alpha \right] \tag{3}$$

subject to the constraints

$$\begin{cases} x^s - \sum_{j=1}^{n} r_j^s \lambda_j = 0, \quad s = 1, \ldots, S \\ \sum_{j=1}^{n} \lambda_j = 1 \\ \lambda_j \geq 0, \quad j = 1, \ldots, n. \end{cases} \tag{4}$$

2.2 Piecewise linear value function

In this subsection we consider the case when in the Kahneman–Tversky value-function $\alpha = 1$ holds. This is the value function discussed in Benartzi and Thaler (1995), Barberis et al. (2001) and Grüne and Semmler (2005). In this case we have

[2] Prospect theory assumes that investors distort probabilities by means of subjective weighting functions. When working with sample data, it is however natural to consider each sample as one equally likely state. Under the assumption of equally probable states of nature, the distortions will not affect optimal portfolio choices, thus the subjective weighting function is dropped in our presentation.

[3] In what follows, we also assume that investors evaluate gains and losses with respect to the zero return.

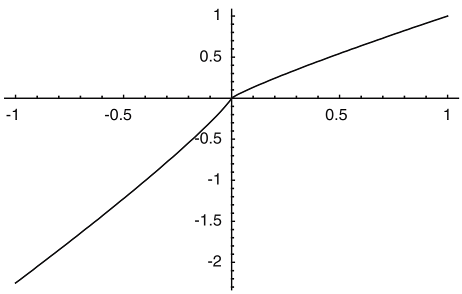


Fig. 1 The Kahneman–Tversky value function with $\alpha = 0.88$ and $\beta = 2.25$

$$v(x) = \begin{cases} x & \text{for } x \geq 0 \\ \beta x & \text{for } x < 0 \end{cases} \tag{5}$$

or in a compact form

$$v(x) = x^+ - \beta x^-.$$

This is a piecewise linear function with a kink at $x = 0$. If $\beta \geq 1$ holds, which we assume in the sequel then $v(x)$ is a concave function. Figure 2 shows this function for $\beta = 2.25$. The sum of concave functions being concave, it follows that $V(x^1, \ldots, x^s)$ is a multivariate concave function. It is a non-linear function which is non-differentiable at points where $x^s = 0$ holds for some s. Figure 3 shows the graph and contour lines of $V(x^1, x^2) = v(x^1) + v(x^2)$ with points of non-differentiability along two lines. For a concave function the upper level sets are concave. In the figure this means that the contour lines, viewed from the south-west corner, are convex curves. Thus, our asset allocation problem belongs to the class of convex optimization problems for which, theoretically, several powerful algorithms exist. Nevertheless, the objective function is non-smooth, which makes the problem difficult to solve numerically for problems with a large number of assets. It is a well-known fact in nonlinear programming that maximizing a piecewise linear concave function subject to linear constraints can be reformulated as a linear programming problem. Thus, fortunately, the asset allocation problem can be equivalently formulated in our case as a linear optimization problem which enables solving large-scale asset allocation problems. For our problem the transformation is particularly simple. We introduce auxiliary variables y^s for representing $(x^s)^+$ and z^s for representing $(x^s)^-$, respectively. Utilizing the fact that $x^s = (x^s)^+ - (x^s)^-$ holds generally, we obtain the following equivalent formulation as a linear programming problem in the $n + 2S$ variables $(\lambda, y^1, \ldots, y^S, z^1, \ldots, z^S)$: Maximize the linear function $\frac{1}{S} \sum_{s=1}^{S} (y^s - \beta z^s)$, subject to the following constraints: we replace the first constraint in the general formulation by $y^s - z^s - \sum_{j=1}^{n} r_j^s \lambda_j = 0$, $s = 1, \ldots, S$, keep the constraints solely involving λ and require $y^s \geq 0$ and $z^s \geq 0$ for $s = 1, \ldots, S$. Note that, besides having obtained a computationally attractive alternative

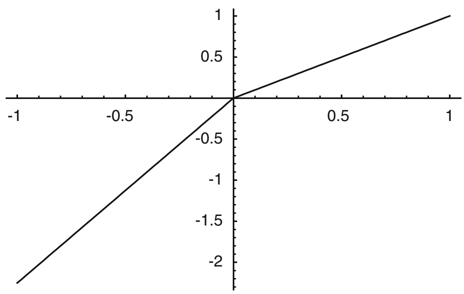


Fig. 2 Piecewise linear value function with $\beta = 2.25$

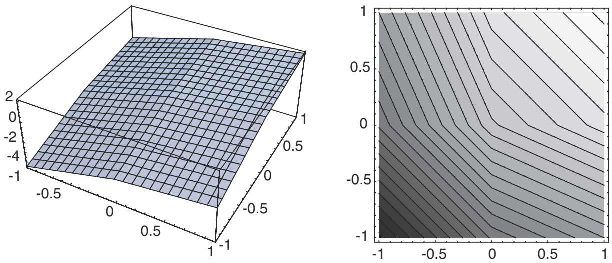


Fig. 3 Graph and contour lines of the sum of two piecewise linear value functions

formulation, the transformation has also eliminated the inconvenient feature of non-differentiability.

2.3 Computational difficulties in the general case

In the case $\alpha \neq 1$, the transformation outlined in the previous section does not work. We will consider the general case with the Kahneman–Tversky parameter settings $\alpha = 0.88$ and $\beta = 2.25$. In the general case, we face two kinds of difficulties from the computational point of view.

On the one hand, the objective function is not differentiable at points where $x^s = 0$ holds for some s. Figure 4 displays $V(x^1, x^2) = v(x^1) + v(x^2)$ where we have, similarly to the piecewise linear case, points of non-differentiability along two lines. Consequently, the problem belongs to the class of non–smooth optimization problems. The size of problems that can be efficiently solved by solvers (implemented algorithms) for this problem class is much smaller than for the smooth case. In addition to this, available algorithms for non–smooth problems essentially rely on the assumption that the objective function is concave. This is not the case for our asset allocation problem. Thus, we arrive at the second kind of numerical difficulty that we face regarding the asset allocation problem. A glance at the graph of $V(x^1, x^2)$ in Fig. 4 confirms that the surface is now curved. Viewing the contour lines in the left-hand-side picture from the south–west corner viewpoint it is clear that some of the contour lines are no more convex curves. This implies that there are some upper level sets

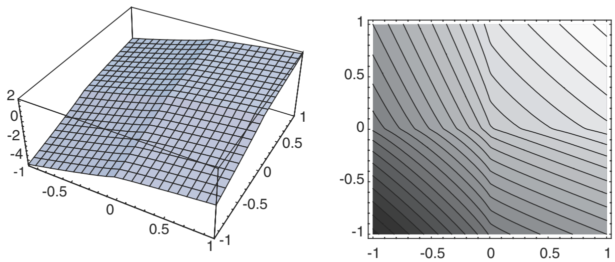
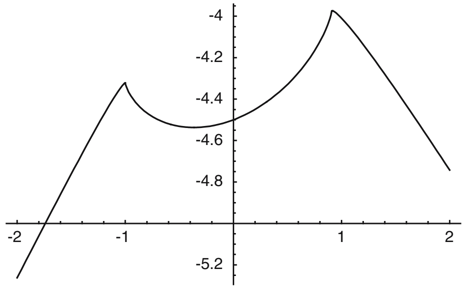


Fig. 4 Graph and contour lines of the sum of two Kahneman–Tversky value functions

Fig. 5 The value function V along a line: $f(\kappa) = V(x^1 + \kappa u^1, x^2 + \kappa u^2)$



that are non–convex sets. Consequently, $V(x^1, x^2)$ cannot be a concave function. Moreover, $V(x^1, x^2)$ is not even quasi-concave, quasi-concavity being defined by the property that all upper level sets are convex sets. For quasi-concavity and for further types of generalized concavity discussed below see, for instance, Avriel, Diewert, Schaible, & Zang (1988).

To confirm the missing quasi-concavity, as suggested by Fig. 4, we proceed by considering the value function along a line, that means, we take $f(\kappa) := V(x^1 + \kappa u^1, x^2 + \kappa u^2)$. Supposing that $V(x^1, x^2)$ is quasi–concave, f would be a univariate quasi–concave function, for any choice of x^1, x^2, u^1, u^2. Let us choose $x^1 = -1$, $x^2 = -1$, $u^1 = 1.1$, and $u^2 = -1$. Figure 5 displays $f(\kappa)$ for $-2 \le \kappa \le 2$. It is immediately clear that the function in the Figure is not quasi–concave (we omit the obvious mathematical proof). Thus, we conclude that $V(x^1, x^2)$ is not a quasi-concave function. On the other hand, pseudo–concavity implies quasi-concavity. Consequently, $V(x^1, x^2)$ is not a pseudo–concave function. This implies that our asset allocation problem may have several local maxima which are not global solutions of the problem.

Let us remark that the value function $v(x)$, being a strictly increasing function, is obviously pseudo–concave. The point is that, unlike for concave functions, summing up pseudo–concave functions does not preserve the pseudo–concavity property. As our example shows, this may even happen in the case when we construct multivariate functions by addition, based on the same univariate pseudo–concave function as in our case $V(x^1, x^2) = v(x^1) + v(x^2)$.

For smaller values of α, the missing quasi-concavity appears more markedly as it can be seen in Fig. 6 which displays V with $\alpha = 0.7$.

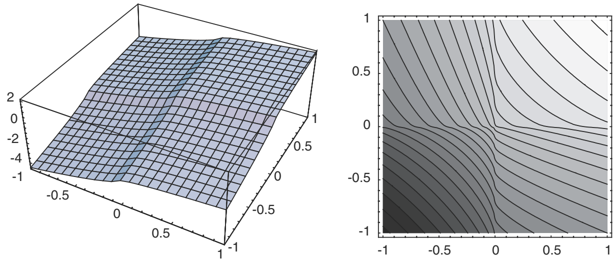


Fig. 6 Graph and contour lines of $V(x^1, x^2)$ with $\beta = 2.25$ and $\alpha = 0.7$

2.4 Overcoming the numerical difficulties

For dealing with non–smoothness we have applied smoothing to the value function in the vicinity of 0, by employing cubic splines. More closely we proceed as follows.

Let $\delta > 0$ and $p(x) = ax^3+bx^2+cx+d$ be a cubic polynomial. The four coefficients of the polynomial are computed from the four equations $p(-\delta) = v(-\delta)$, $p'(-\delta) = v'(-\delta), p(\delta) = v(\delta), p'(\delta) = v'(\delta)$, thus ensuring that both the function values and the first derivatives of p and v are equal, at both endpoints of the interval $[-\delta, \delta]$. The value function is subsequently replaced by the smoothed value function

$$v_\delta(x) := \begin{cases} p(x) & \text{if } x \in [-\delta, \delta] \\ v(x) & \text{otherwise} \end{cases} \tag{6}$$

leading to the following approximation of the objective function of the asset allocation problem

$$V_\delta(x^1, \dots, x^s) := \frac{1}{S} \sum_{s=1}^{S} v_\delta(x^s). \tag{7}$$

Denoting the maximal approximation error over the interval $[-\delta, \delta]$ by ε, we have

$$V_\delta(x^1, \dots, x^s) \leq \frac{1}{S} \sum_{s=1}^{S} (v(x^s) + \varepsilon) = \frac{1}{S} \sum_{s=1}^{S} v(x^s) + \varepsilon = V(x^1, \dots, x^s) + \varepsilon$$

and similarly we get $V_\delta(x^1, \dots, x^s) \geq V(x^1, \dots, x^s) - \varepsilon$. Thus, replacing our objective function with $V_\delta(x^1, \dots, x^s)$ results in an ε–optimal solution, with respect to the true optimal objective function value. Choosing $\delta > 0$ small enough, $\varepsilon > 0$ can be made arbitrarily small in theory. In practice, $\delta = 0.00001$ turned out to be small enough.

For solving the smoothed problems we have employed the general-purpose solver Minos 5.4 (Murtagh & Saunders, 1978, 1995), designed for smooth nonlinear programming problems.

The second difficulty, the possible presence of several local optima, has been dealt with as follows. First N starting points have been randomly generated on the unit simplex $\{\lambda \mid \sum_{j=1}^{n} \lambda_j = 1,\ \lambda_j \geq 0\ \forall j\}$, according to the uniform distribution over the unit simplex. For this we employed an algorithm of Rubinstein (1982). The following procedure has been repeated N times:

- n pseudo–random numbers $z_1, \ldots, z_n$ are generated according to the uniform distribution on $[0, 1]$.
- These are transformed as $y_j = -\log(z_j)\ \ \forall j$, corresponding to the exponential distribution with parameter 1.
- Finally, the normalization $\lambda_j = \dfrac{y_j}{\hat{y}}$ with $\hat{y} := \sum_{j=1}^{n} y_j$ results in a random vector λ, corresponding to the uniform distribution over the unit simplex.

Subsequently, the solver Minos has been started up from the N starting points in turn, resulting in N (locally optimal) asset allocations with corresponding optimal objective values $\tilde{V}_1, \ldots, \tilde{V}_N$. The allocation with the highest $\tilde{V}_j$ value has been chosen as the solution of the problem. The optimization algorithm outlined above belongs to the class of multistart random search methods, see Törn and Zilinskas (1989).

Obviously, the quality of the solution largely depends on the proper choice of N. In practice, we took an initial N and started up the solver. This has been repeated with increased N till the solution did not change.

2.5 Outlook on planned developments of the method

The next step in the algorithm development, still based on the general-purpose solver Minos, will be the inclusion of adaptive elements into the procedure for selecting starting points. This procedure we plan to design in the spirit of the adaptive grid method of Grüne and Semmler (2003). On the long range we plan to develop a special–purpose algorithm for finding the global optimum, based on the structure of the problem.

3 Asset pricing applications

Replacing the piecewise linear value function with the piecewise power function is important because it does incorporate risk taking for losses and risk aversion for gains, as it is robustly observed in laboratory findings. However, due to the loss in quasi-concavity it is not clear a priori which asset pricing implications are introduced this way. In this section we check whether the approximation of the piecewise power value function by a piecewise linear function can be justified. On standard data for a broad stock and a broad bond index, as it can be found on the homepage of Kenneth French, we find that introducing asymmetric risk aversion does bring the parameter values of the representative asset pricing model closer to those found in the laboratory. Hence by doing so we gain on both sides: the representative agent has a richer behavior and its parameter values are closer to those observed in the laboratory. Thereafter, we compare both the piecewise linear and the piecewise power function on a richer set of assets including the standard Fama and French size and value portfolios. It is found that the resulting asset allocations are still similar but they are much different to the optimal mean-variance portfolio. The data we use has the following summary statistics (See Appendix): The annual real equity premium is about 6.4% but the equity index is also much more volatile than the bond index. Both indices also differ in kurtosis and skewness. The size and the value portfolios show the well known size and value effect, i.e., the high excess return of small cap to large cap and of value to growth stocks. We set the reference point for the net-return data we used equal to 0, assuming that obtaining the risk free rate is already seen as a success of the investment.

To analyze the Equity Premium Puzzle we adopt the following methodology. Working with the representative consumer model we cannot use the standard Euler equation approach because due to the lack of quasi-concavity it may not describe the sufficient condition for the optimal asset allocation. Instead we use our algorithm to solve for globally optimal prospect theory portfolios as described above. The task is to find the parameters α and β of the value function such that the representative consumer holds the stock and the bond index in proportion to their market capitalization. For the relative size of the bond and the stock market we follow Bandourian and Winkelmann (2003) who estimate the bond equity proportion to be approximately 50:50. First we investigate the piecewise linear value function on the bond and the stock market index described above. After some iterations we find a loss aversion of about 2.353. Figure 7 shows that the optimal asset allocation is not perfectly robust around this value but the bond stock split of the market portfolio is also only an approximation. Then we investigate the piecewise power value function for a loss aversion as found in the laboratory (2.25) and search for a risk aversion in order to also get the 50:50 split. The value found is about 0.894 (see Fig. 8). Again around this value the optimal asset allocation shows a jump. The size of the jumps for the piecewise power value function are however not larger than those for the piecewise linear function.

Finally, we fix these parameter values and compare the portfolio choice of the two prospect theory functions with that of a mean-variance investor maximizing the ratio of mean to variance. We find for the piecewise linear value function[4]: ME1 = 38.19%, BM8 = 14.37%, BM9 = 47.44% and for the piecewise power: ME1 = 36.07%, BM8 = 12.63%, BM9 = 51.29% while we find the optimal mean-variance portfolio (maximal ratio of mean to standard deviation) to be: ME1 = 4.61%, BM5 = 4.71%, BM8 = 19.23% and Bond = 71.44%. Hence the two prospect theory optimization problems lead to quite similar results that are however quite different to the mean-variance portfolio.

[4] ME denotes the size portfolios. It reads as market to equity portfolio. BM denotes the value portfolios. It reads as book to market portfolio.

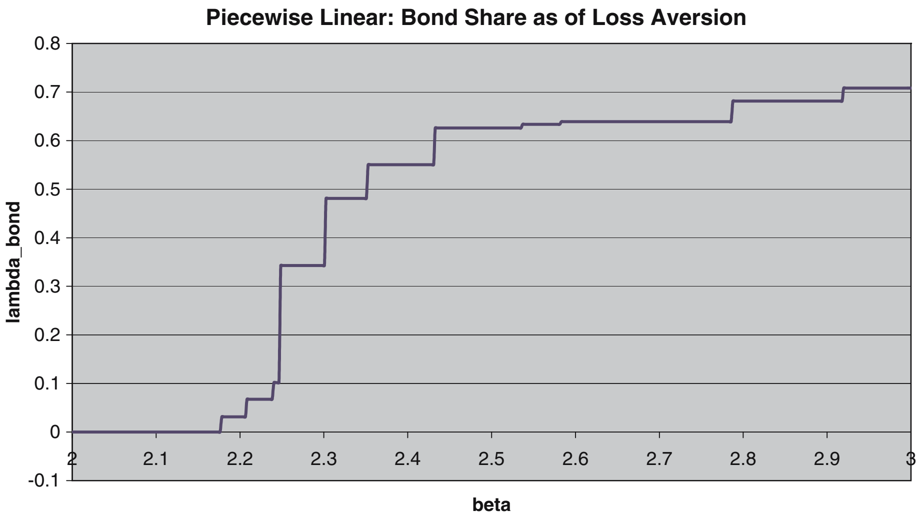


Fig. 7 Bond share as of loss aversion for the piecewise linear value function

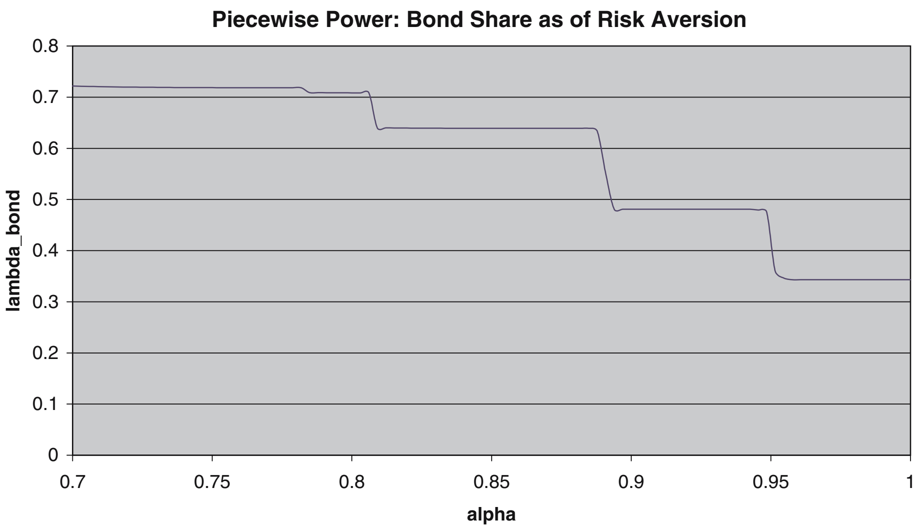


Fig. 8 Bond share as of risk aversion for the piecewise power function

4 Conclusion

We developed an algorithm to compute asset allocations for Kahneman and Tversky's (1979) prospect theory. An application to benchmark data as in Fama and French (1992) shows that the equity premium puzzle is resolved for parameter values similar to those found in the laboratory experiments of Tversky and Kahneman (1992). While previous studies like Benartzi and Thaler (1995), Barberis et al. (2001), and Grüne and Semmler (2005) only used myopic loss aversion to explain the equity premium puzzle our paper extends this explanation of the equity premium puzzle by incorporating asymmetric risk aversion.

The introduction of asymmetric risk aversion bears some considerable cost in terms of computational efforts but it comes at no cost for the economic result. To the contrary: the values found are even more similar that those found in the laboratory, however both value functions considered lead to jumps in the optimal asset allocation– also in the area of the values found on the data. The challenge for further research is to find value functions that account for loss aversion and asymmetric risk aversion but that show more robustness when optimized over realistic data. The piecewise exponential value function by De Giorgi Hens and Levy (2004) may be a candidate also for this matter.

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Appendix

Table 1 shows descriptive statistics (average, standard deviation, skewness, excess kurtosis and max and min) for the annual real returns of the value–weighted CRSP all–share market portfolio, the intermediate government bond index of Ibbotson and the size and value decile portfolios from Kenneth French' data library. The sample period is from January 1927 to December 2002 (76 yearly observations).

Table 1 Descriptive statistics

	Avg.	Stdev.	Skew.	Kurt.	Min	Max
Equity	8.59	21.05	−0.19	−0.36	−40.13	57.22
Bond	2.20	6.91	0.20	0.59	−17.16	22.19
Small	16.90	41.91	0.92	1.34	−58.63	155.29
2	13.99	37.12	0.98	3.10	−56.49	169.71
3	13.12	32.31	0.69	2.13	−57.13	139.54
4	12.53	30.56	0.46	0.83	−51.48	115.32
5	11.91	28.49	0.44	1.60	−49.57	119.40
6	11.65	27.46	0.31	0.61	−49.69	102.17
7	11.09	25.99	0.30	1.14	−47.19	102.06
8	10.15	23.76	0.29	1.19	−42.68	94.12
9	9.63	22.33	0.02	0.46	−41.68	78.15
Large	8.06	20.04	−0.22	−0.52	−40.13	48.74

Table 1 Continued

	Avg.	Stdev.	Skew.	Kurt.	Min	Max
Growth	7.84	23.60	0.02	−0.64	−44.92	60.35
2	8.77	20.41	−0.27	−0.27	−39.85	55.89
3	8.52	20.56	−0.10	−0.47	−38.00	51.90
4	8.25	22.49	0.49	2.39	−45.02	96.33
5	10.29	22.82	0.36	1.92	−51.55	93.77
6	10.05	23.04	0.19	0.63	−54.39	73.57
7	11.00	24.73	0.18	1.22	−51.13	97.91
8	12.82	27.01	0.67	1.95	−46.56	113.53
9	13.71	29.08	0.56	1.85	−47.42	123.72
Value	13.32	33.05	0.43	1.40	−59.78	134.46

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